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THE RATE OF THE M43A3 MECHANICAL TIME FUZE, AND ITS DIFFERENTIAL EFFECTS

by

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Abstract

The rate of the M43A3 mechanical time fuze, during flight, depends slightly upon driving torque, markedly upon the rate of spin, and trivially upon the fuze temperature. There is a small elastic strain effect, arising from the straining of the mechanism between the timing disk and the escapement. The amounts of these effects, determined experimentally, are applied to compute the relation between time to burst and fuze setting on the 3-inch A.A. gun M3, the 90mm A.A. gun M1, and the 75mm aircraft gun M2. The differential effects of changes in muzzle velocity, air density, and angle of departure are also determined. Preliminary computations are described relating to the 4.7-inch A.A. gun. The derivation is described of the time-to-burst, fuze-setting relation for the 3-inch A.A. gun M3, included in the fuze specifications.

It is pointed out that the manufacturer can change the zero-point slightly by changing the initial elastic torque in the fuze, which should therefore be closely controlled. He can also readily modify the rate to which the fuze is regulated, so as to bring about a certain type of change in the time-to-burst, fuze-setting relationship. He cannot readily modify the dependence of rate on spin.

A new type of escapement is mentioned, that would necessitate an extensive re-design of the fuze, but which would result in nearly complete independence of the rate of the fuze and the rate of spin. It is expected that such a fuze would behave similarly in all guns, its time to burst being equal to its fuze setting.

Introduction

In a letter dated September 2, 1941, and reproduced at the end of this report, the Chief of Ordnance ordered the Ballistic Research Laboratory to prepare a program of firings in order to determine the time-to-burst, fuze-setting relation and the differential effects for the M43A3 mechanical time fuze. The results of the firings are analyzed in the following report.

The writer makes most grateful acknowledgements to Mr. R. H. Kent, of the Ballistic Research Laboratory, for his valuable advice; and to Mr. J. A. Middlemiss, of the Frankford Arsenal, for much information.

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1. The Mechanical Principles Underlying the Operation of the Fuze.

We describe the construction and operation of the M43A3 mechanical time fuze to an extent necessary for the subsequent discussions. The time fuze consists essentially of a clock-work mechanism, driven by centrifugal force acting upon two gear wheels. These gears, called the "centrifugal gears", have their centers of mass at some distance from their pivots. The spin of the projectile causes a centrifugal force to act through each center of mass, producing a moment tending to rotate each of the gears about its pivot. This torque is applied to a shaft, carrying the "timing disk", through teeth on the shaft that engage with teeth on the centrifugal gears. The shaft is thereby caused to rotate, carrying with it the timing disk. The rate of rotation of the shaft is controlled by an escapement similar, generally, in its purpose and arrangement, to the escapement of a watch. The shaft is joined to the escapement by a system of gearing. The timing disk has a slot in its circumference, and after the disk has rotated sufficiently to allow an arm to fall into this slot, a detonator is fired. Before the round is fired, the fuze is set by rotating the timing disk sufficiently on its shaft, on which the disk has a slipping fit.

The fuze will fire at all settings between a lower limit of about 2 seconds, and 30 seconds.

When the round is fired, a special unlocking device releases the escapement, and the rotation of the projectile then causes the fuze mechanism to run. To insure the prompt starting of the mechanism, the escapement is driven backward, during manufacture, thereby causing the mechanism to run backward, until a centrifugal gear reaches the limit of its motion and jams. Further backward motion of the escapement strains the gears and shafts elastically, until a predetermined torque has been reached, and then the escapement is locked. This elastic strain causes the fuze mechanism to start to run the instant the escapement is unlocked. After firing, the fuze runs until the timing disk has rotated sufficiently to allow the arm to fall into its slot, at which time the firing mechanism operates and the projectile is exploded.

2. The Rate of the Escapement.

2a. General

The escapement is very similar to the cylinder escapement used on some watches. The balance wheel has a stiff linear hair-spring[1], and its pallets (approximately

[1] Historically, it may be of interest to recall that the first hair spring used in a watch, during the sixteenth or seventeenth century, was linear — a pig's bristle.

portions of a single cylindrical surface) engage directly with the teeth on the escape wheel. The axis of the balance wheel coincides with the longitudinal axis of the fuze and of the projectile. The period of oscillation of the balance wheel, and hence the rate of the escapement, depends primarily upon its moment of inertia and upon the stiffness of the hair-spring. The stiffness of the hair-spring may be defined as the moment required to hold the balance wheel, against the resistance of the hair-spring, at unit angular displacement from the equilibrium position. After manufacture, the moment of inertia of the balance cannot be altered for it contains no screws. The stiffness of the spring, however, may be adjusted by increasing or decreasing the distance between its supports, which are provided with micrometer adjustments. The escapement is adjusted to a pre-determined rate, at the time of manufacture, by the use of the micrometer adjustments.

If the balance wheel were acted upon by no forces other than those due to the hair-spring, and if the moment of the forces applied to it by the spring were exactly proportional to the angular displacement of the balance wheel from its equilibrium position, then the period of the balance wheel would be completely independent of the amplitude of its vibration. Actually, however, the distance between the supports of the hair-spring is rather short, so that the hair-spring experiences a finite and not infinitessimal bending; and thus the moment can not be expected to be exactly proportional to the displacement. The existence of various free periods in the hair-spring causes further departures from proportionality during the actual running, with the result that the escapement cannot be exactly isochronous, and should have a rate dependent on the amplitude of its vibration, which in turn depends upon the driving torque.

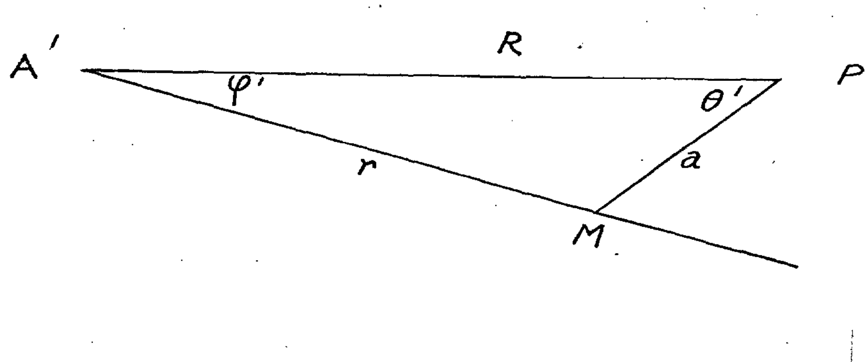
Even when "free", the fuze escapement should not be as nearly isochronous as the escapement of a watch, having a relatively lighter and much longer hair-spring; but the fuze escapement is free only for a very small portion of its motion. Most of the time the pallets are in contact with the teeth of the escape wheel, and thus one must expect the rate of the fuze to depend directly upon the torque that is applied to the escape wheel, through the gear train, by the centrifugal gears. The torque depends in turn upon the rate of spin of the projectile, and also upon the length of time the fuze has run; and thus the rate of the fuze should depend upon the projectile's instantaneous rate of spin and upon the time that has elapsed since starting. Further, the rate of spin of the projectile influences the tension of the hair-spring through centrifugal force, acting on each element of the hair-spring's length. Increasing the rate of spin increases the tension of the hair-spring, increases its stiffness, and therefore increases the rate of the escapement. Finally the rate of the fuze must be influenced by its temperature, which influences the stiffness of the hair-spring and the moment of inertia of the balance wheel.

To summarize, the rate of the fuze can be influenced (1) by the driving torque. Changing the torque (a) changes the amplitude of oscillation of the balance, thereby bringing in the isochronism error of the free escapement and (b) changes the forces applied by the escape-wheel teeth to the pallets, thereby perturbing the motion of the balance and influencing its rate. The driving torque depends upon the rate of spin, and also upon the time the fuze has run. In addition, the rate of the fuze should depend directly (2) upon the rate of spin (aside from the indirect effect, of the spin upon the driving torque and of the driving torque upon the rate) because of the effect of the spin upon the stiffness of the hair-spring. Finally (3), the rate should depend upon the temperature of the fuze.

2b. The Effect of the Driving Torque Upon the Rate of the Fuze.

Experiments performed by the writer showed an appreciable dependence of the rate upon the driving torque, when the rate of spin was zero. The torque was applied statically, by suspending known weights from a cord wound around a drum mounted in place of the timing disk. Increasing torque resulted in a slowing of the fuze, a result confirmed orally to the writer by Mr. J. A. Middlemiss of the Frankford Arsenal, who had made the same observation. For reasons which are not clear, the situation when the fuze is spinning, as in flight, differs. Then a considerable variation of torque at constant spin has only a very slight effect upon the rate, although such effect as there is still corresponds to a faster rate for a lesser torque.

In the diagram below, A' is the center of rotation; P is the pivot of a centrifugal gear; R is the distance A'P; M is the center of mass of the centrifugal gear; r is the distance A'M; a is the distance MP; φ' is the angle PA'M; θ' is the angle A'PM.



Denote by ω the constant angular velocity of spin of the projectile, and denote the mass of the centrifugal gear by M. We wish to compute the torque produced by the centrifugal gear. When the fuze is running, the angle θ' is changing at an almost exactly uniform rate, so that the centrifugal gear is rotating at a rate, slightly different from ω, but nevertheless constant. Since the gear has no angular acceleration, the resultant of the external forces acting on the gear must pass through M. The acceleration of M is $\omega^2 r$, in the direction MA'. In absolute units, the resultant of the external forces acting on any body is equal to the mass of the body multiplied by the acceleration of the center of mass of the body. Hence the resultant of the external forces acting on the centrifugal gear is equal to $M\omega^2 r$, and acts along the line MA'. Action and reaction being equal and opposite, the centrifugal gear must apply a force $M\omega^2 r$, whose line of action is A'M, to the rest of the mechanism. The moment produced by the gear about its pivot P, and therefore its torque, is the resultant force times the perpendicular distance from P to A'M, and the product is

$$\text{Torque} = M\omega^2 r\ R \sin \varphi'$$

or

$$\text{Torque} = M\omega^2 a\ R \sin \theta' \qquad (1)$$

since

$$\sin \varphi'/a = \sin \theta'/r.$$

When the fuze is running at constant spin, the only quantity that changes in the right-hand member of (1) is the factor $\sin \theta'$, and thus the torque at any place in the mechanism varies as $\sin \theta'$. In particular, the torque produced by the gear in the timing-disk shaft is 7/30 of the torque given by (1), since there are seven teeth on the timing-disk shaft and thirty to a complete circumference of a centrifugal gear. Denote by θ the time (fuze time, equal to 32 seconds times the fraction of a revolution completed by the timing disk, there being 32 "seconds" to a complete revolution) the fuze has run since starting; then

$$\theta' = e + (7/30)(360/32)\ \theta \text{ degrees}$$
$$= e + 2^\circ.625\ \theta$$

where $\underline{e}$ is some constant. Hence the driving torque in the timing disk shaft due to a centrifugal gear is given by

$$\text{Torque} = M\omega^2 aR \sin(e + 2^\circ.625\ \theta)\ (7/30)$$

The other gear engages with the odd-toothed timing-disk shaft half a tooth differently from the gear already considered, so its contribution to the total torque is

$$M\omega^2 aR \sin(e \pm 6^\circ + 2^\circ.625\ \theta)\ (7/30)$$

where either the + or the - sign is used according to the nature of the engagement. In either case, the total torque, T_g, in the timing-disk shaft is the sum of the contributions of the two gears, and is

$$T_g = 2M\omega^2 aR\ (7/30)\cos 3^\circ \sin(e \pm 3^\circ + 2^\circ.625\ \theta). \qquad (2)$$

From data supplied to the writer by the Frankford Arsenal, the constants have been evaluated and the result is

$$T_g = 0.5318\omega^2\ f(\theta) \qquad (3)$$

inch-pounds on the timing-disk shaft, when ω is in hundreds of revolutions per second (in earlier equations, ω has been in radians per second). The function $f(\theta)$ is given by

$$f(\theta) = \sin\ (36^\circ.474 + 2^\circ.625\ \theta) \qquad (4)$$

When the fuze is fired with a muzzle velocity of 2700 feet per second from a 3-inch gun having a twist of rifling of one turn in 40 calibers, then just at the muzzle $\omega = 2.7$. This value of ω is also the value at which the fuzes are regulated by the Frankford Arsenal; under such conditions the coefficient $0.5318\omega^2$ has the value 3.877 inch-pounds. The following table gives the values of $f(\theta)$, and the total torque in the timing-disk shaft at 270 revolutions per second, for values θ of the run of the fuze up to the limit of 30 seconds, at which the fuze stops:

Time run, in fuze seconds θ	Torque in inch-lbs at 270 RPS	Ratio of torque to torque after 20 second's running, at constant spin $f(\theta)$
0	2.30	.594
5	2.95	.762
10	3.45	.889
15	3.76	.970
20	3.88	1.000
25	3.79	.978
30	3.51	.905

It appears that the torque is least at first, increases by 67% up to the value at twenty seconds, and then diminishes somewhat near the 30-second limit of the fuze's operation.

The M43A3 fuze is adjusted during manufacture to run at a rate of approximately 172.839 vibrations of the balance wheel per second, at room temperature (about 70°F), and at a spin of 270 revolutions per second. From the gear ratios, 172.265625 vibrations correspond exactly to one fuze second (1/32 of a revolution of the timing disk). The adjustment is not exact, and cannot be since the rate varies during the run.

The Frankford Arsenal has kindly supplied the writer with the results of certain experiments which show the variation of rate, during runs at constant spin. Twenty-six fuzes were rotated at 270 revolutions per second, at room temperature, and for each of them the fuze error $t-\theta$ was recorded as a function of t. Here t is the time in seconds since starting, θ is the fuze time in seconds. The averages of the twenty-six results appear in the second column below:

t sec	$t-\theta$ sec	$0.000045\,t$ sec	Difference sec
0	0	0	0
5	-0.0039	0.0002	-0.0041
10	-0.0038	0.0004	-0.0042
15	-0.0020	0.0007	-0.0027
20	+0.0003	0.0009	-0.0006
25	+0.0021	0.0011	+0.0010
30	+0.0057	0.0014	+0.0043

In the third column are listed values of $0.000045\,t$, and in the fourth column are the differences $t-\theta-0.000045\,t$. The largest deviation from uniform running thus corresponds to only 0.0043 seconds, although the torque varied from 2.30 inch-pounds at 0 seconds to 3.88 inch-pounds at 20 seconds, in the timing-disk shaft. The deviations are so small that for practical purposes, the variation of rate with torque, at constant spin, may be ignored. The standard deviation of a single fuze[1] under firing conditions is about twenty-five times larger, varying from about $0^s.05$ at 2 seconds to about $0^s.20$ at 30 seconds.

The writer, by a careful analysis of the preceding experimental results at constant spin, has found that an increase of one inch-pound, in the torque in the timing-disk shaft at a constant spin of 270 revolutions per second, makes the fuze run slower by 111 parts in 100,000. This refers to the change in $\dot{\theta}$, the time derivative of θ; the observed values of the fuze error, $t-\theta$, depend on the time intergral of $\dot{\theta}$. In computing the deviation from uniform running, uniform running has been taken to correspond to a losing rate of 0.000045 seconds per second, in order to make the greatest positive and negative deviations equal. We are here concerned with changes of rate, rather than with departures from a zero rate, for the latter departures are influenced by the regulation of the escapement.

[1] When manufactured by the Frankford Arsenal.

2c. The Effect of the Rate of Spin Upon the Rate of the Fuze.

Apart from firings, there have been no experimental determinations of the effect of spin upon rate. In addition to the very small effect, of the rate of spin upon the rate of the fuze, that arises from the dependence of the rate upon the torque and of the torque upon the spin, there is also the effect of the spin upon the stiffness of the hair-spring, which in turn influences the rate. The general nature, however, of the dependence of the rate, $\dot{\theta}$, upon spin, ω, is known through the following considerations. The rate must clearly be a function of ω; and it is known from firings that over the range of spins encountered in practice, the rate varies by only one or two parts in ten. Between the lowest spin at which the fuze will run at all, and the highest spin that it can stand without mechanical failure, the rate must, moreover, be a smooth function of ω. Under these circumstances, the rate can therefore be represented by a power series in ω. Further, the rate must be the same at a spin $+\omega$ as it is at a spin $-\omega$, so that no odd powers of ω can appear in the series. Hence it is easy to see that the total effect of the spin upon the rate must be of the type

$$\dot{\theta} = b' + c'\omega^2 + e'w^4 + g\omega^6 + \ldots$$

where b', c', g,... are constants. The torque varies as ω^2, so that the effect, of the spin upon the rate, that arises through the torque contributes only to the term $c'\omega^2$. The writer has examined the stiffness terms, and finds that $\dot{\theta}$ must be an infinite series in ω^2; and thus the total effect of the spin strictly demands an infinite series for its exact representation. The dependence of the rate upon the spin, however, for practical purposes may be represented by a polynomial in ω^2, provided that the polynomial represents the rate accurately enough for practical purposes, for values of ω encountered in practice. In particular, it may be found that the first two terms

$$\dot{\theta} = b' + c'\omega^2 \qquad (6)$$

will yield results agreeing with observation within the observational uncertainties, and we shall therefore tentatively adopt (6) for the form of the relation. If (6) satisfies the observations of times to burst satisfactorily, with values of $\underline{b}'$ and $\underline{c}'$ found from the observations, then it will be impossible to find the higher coefficients from observations of that accuracy, and unnecessary as well. Equation (6) will in fact be found to represent the observations reasonably well, within the working range of ω.

The value of $\underline{c}'$ found experimentally, from firings, is about +0.01 when ω is expressed in hundreds of revolutions per second. This will be shown in a later section. The contribution to $\underline{c}'$ that arises from the effect of spin upon torque and of torque upon rate may be obtained from the result described in section 2b, and is

$$(-.00111)\ (.5318)\ f(\theta) = -0.0005$$

approximately. Thus the total effect of spin upon rate is twenty times larger than its effect through the torque, and of opposite sign. We conclude that the main effect of spin upon rate is probably through the "stiffness" of the hair-spring, rather than through the torque. Increasing the rate of spin increases the tension in the hair-spring, increases its stiffness, and increases the rate at which the fuze runs.

2d. <u>The Effect of Temperature Upon the Rate of the Fuze.</u>

An increased temperature alters the elastic properties and dimensions of the hair-spring, and changes its stiffness. An increased temperature enlarges the uni-metallic balance wheel, and increases its moment of inertia. If made of any usual metal, the hair-spring becomes weaker with increasing temperature, resulting in an over-all slowing of the fuze at higher temperatures, and a quickening at low temperatures. With an "Elinvar" hair-spring, the stiffness is nearly independent of temperature; if the balance were made of "Invar", there would be practically no temperature dependence of the rate. The balance, however, is not made of Invar; its exact coefficient of thermal expansion is unknown, but is probably of the order of 5×10^{-6} per degree Fahrenheit. With an Elinvar hair-spring, the rate of the fuze should therefore diminish by about five parts in one million for each degree Fahrenheit that the temperature is lowered.

Twenty-one fuzes were run by the Frankford Arsenal at 197°F, and again at -50°F. The results have very kindly been communicated to the writer. On the average, the twenty-one fuzes were slower by 0.073 ± 0.011 (s.d.) seconds after running 30 seconds at the high temperature than at the low. Thus the rate increases on the average by about 9.8 ± 1.5 parts in one million for each degree Fahrenheit that the temperature is lowered. The observed result is thus of the same order of magnitude as the expected result, and such difference as there is can be attributed to a slight temperature dependence of the hair-spring, which is not made of Elinvar, but of a similar alloy. In any case, the temperature effect is trivial. The effect of a difference in temperature of even as much as one hundred degrees Fahrenheit is to alter the running of the fuze by only $0^{s}.03$ at thirty seconds, which is small compared with the standard deviation of the fuze under firing conditions. For practical purposes, the dependence of the rate of the fuze upon the temperature of the fuze can be completely ignored.

2e. The Rate of the Fuze. Summary.

It has been found that the dependence of the rate of the fuze upon the driving torque is small enough to be ignored, and also that the dependence of the rate upon the temperature of the fuze is small enough to be ignored. The only important variation of the rate of the fuze, after it has been regulated, is with the rate of spin of the projectile. This dependence probably is introduced by the hair-spring, which must be stiffer at high rates of spin than at low rates of spin. The amount of the dependence will be investigated in a subsequent section.

3. Elastic Strain Effect

The long system of gearing between the timing disk and the escapement introduces appreciable elasticity into the mechansim. As has been mentioned, use is made of this elasticity to establish an initial torque in the fuze during manufacture. The escapement is run backward by a suitable tool, equipped with a ratchet device so that after a predetermined torque is reached the tool slips and no further backing up of the escapement occurs. Then the escapement is locked. At the beginning of the backing process, the timing disk runs backward until its shaft is stopped from further backward rotation by its teeth, which jam against the untoothed portion of the circumference of one of the centrifugal gears. Afterwards, further backing requires an increasing torque, until the backing tool slips and further backing-up stops.

The writer has experimentally determined the moment of torsion of the gear train, between the escape wheel and the point on the timing disk shaft where the centrifugal gears engage. The centrifugal gears and timing disk were removed from a fuze. A small mirror was cemented to the timing-disk shaft, at the toothed portion that normally engages with the centrifugal gears, so that the mirror was parallel to the shaft. A drum of known radius was firmly attached to the timing-disk shaft in place of the timing disk. No other change was made in the fuze, which was then clamped with its axis horizontal. The escapement was locked, and known weights were suspended from a string wrapped around the drum, so as to apply a known torque to the timing-disk shaft. The resulting elastic strain was measured by observing the angle through which a beam of light, projected from a small lantern on to the little mirror, was deflected on a screen after reflection by the mirror. The distance from the mirror to the screen was 40 1/8 inches, and the deflection of the spot was measured in units of 1/20 inches. Thus a motion of the spot of one unit corresponded to a rotation of the light beam of 0.001246 radians, and to a rotation of the mirror of half this, or 0.000623 radians--or 2.14 minutes of arc. In practice, when the fuze is running, the torque is applied not to the outer end of the timing-disk shaft, but to the toothed portion where the mirror is placed. No straining of the timing disk shaft occurs between the toothed portion and the outer, timing-disk end. In the experiment, the torque was applied to the outer end of the timing-disk shaft, and had the mirror been placed at this end of the shaft it would have yielded too large a twist by the amount of twist that occured between the outer end of the shaft, and the toothed portion where the torque is applied during normal running. However, by placing the mirror at the toothed portion of the shaft, the twist of the shaft between the toothed portion and the outer end was automatically eliminated from the measures. Thus the twist measured by the light-spot, for any applied torque, was precisely equal to the twist of the fuze mechanism, between the escapement and the timing disk, produced by the same torque during normal running. A twisting of the timing disk through 0.000623 radians corresponds to 0.00317 fuze seconds. It was found that the displacement of the light-spot was proportional to the applied torque, and that with the escapement clamped a torque of one inch-pound produced a rotation of the timing disk of $0^s.0162$. The individual values for three fuzes were $0^s.0163$, $0^s.0155$, and $0^s.0167$, of which $0^s.0162$ is the mean.

The elasticity of the mechanism has the consequence that any change in torque causes a change in the relationship of the timing disk to the escapement. During flight, the changing torque (1) of the centrifugal gears introduces a gradually changing relationship. And just after the round is fired, (2) the rapid diminution of the initial torque, in the first fraction of a second of the escapement's running, introduces a sudden change in the relationship of timing disk to escapement which in turn introduces a dependence of the error of the fuze, $t-\theta$, upon the amount of the initial torque to which the mechanism has been adjusted during manufacture.

(1) During flight, the driving torque, T_g, on the timing-disk shaft is given, in inch-pounds, by equation (3). Thus the effect on the timing disk is

$$(0.5318)\ (0.0162)\omega^2 f\ (\theta) = 0.0086\omega^2 f(\theta)$$

seconds. The greater the torque, the sooner the fuze fires.

Thus a term

$$-0^{s}.00086\omega^{2}f(\theta) \qquad (7)$$

is introduced by elastic strain into the fuze error, $t-\theta$. This term will be taken into account later in the present report.

(2) As has been mentioned, an initial torque is established during manufacture by backing up the escapement and locking it. As soon as the round is fired, the escapement starts to run. The torque, T_g, supplied by the centrifugal gears is given by equation (3), and just after firing, from the 3-inch AA gun M3, with a muzzle velocity of 2700 feet per second and a twist of rifling of 1/40, the torque T_g is 2.30 inch-pounds. The elastic torque, incorporated in manufacture, is greater than this--in all events, for the Frankford Arsenal fuzes. Consequently a diminution of torque occurs, very rapidly, from the high elastic value to the lower value that can be maintained by the centrifugal gears; and with this diminution there occurs an untwisting of the mechanism between the escapement and the timing disk. The untwisting introduces a term into the fuze error, $t-\theta$, equal to

$$0^{s}.0162\,(T_i-T_g). \qquad (8)$$

There T_i is the initial elastic torque, in inch-pounds, in the timing-disk shaft. The part of the term $-0^{s}.0162T_g$ is merely the term (7), already considered. The part $0^{s}.0162T_i$ expresses the dependence of the fuze error, $t-\theta$, upon the initial torque T_i.

The writer has been informed by Mr. J. A. Middlemiss, of the Frankford Arsenal, that the ratchet device used in adjusting the initial elastic torque, in the fuzes manufactured by the Arsenal, establishes a torque of from 0.45 to 0.60 inch-pounds in No. 2 shaft. The gear ratio between No. 2 shaft and the timing-disk shaft is 21:1. Thus the Arsenal puts an initial torque of from 9.45 to 12.60 inch-pounds into the timing-disk shaft by its ratchet device, of which the average effect upon the fuze error, $t-\theta$, is about 0.18 seconds. The average effect cannot be observed; it contributes only to the zero-point error of the fuze. A change, however, from 0.45 to 0.6 should increase the time to burst, at all fuze settings, by 0.05 seconds. Thus the variation in the initial torque is important, and the preceding results indicate that if the fuzes are to function accurately, the initial torque should be carefully controlled--perhaps more carefully than at present. A torque tool with a spring and an indicating pointer, on the principle of the larger wrenches used for inserting spark-plugs in automobile engines, would avoid the discontinuities of a ratchet and provide a more accurate control.

4. The Error of the Fuze

From the preceding discussion, it appears that we must take into account the effect of spin on rate, and the elastic strain effect. Integrating equation (6), for the effect of spin on rate, with respect to the time we find

$$\theta = a' + b't + c' \int_0^t \omega^2 dt$$

where a' is a constant of integration, and where b' is related to the rate of the fuze at a standard spin. We must also include the elastic strain effect, given by (7), and then we get

$$\theta = a'' + b't + d'\omega^2 f(\theta) + c' \int_0^t \omega^2 dt$$

where a'' is a new constant, and d' has the value 0.0086 when ω is in units of one hundred revolutions per second. These units will be used hence-forth. Accordingly, the fuze error, $t - \theta$, is given by

$$t - \theta = - a'' - b't - d'\omega^2 f(\theta) - c' \int_0^t \omega^2 dt \qquad (9)$$

Before we can use this formula, we must describe the spin ω in terms of the time, t.

The differential equation satisfied by the spin, ω, was suggested by Fowler, Gallop, Lock and Richmond[1] to be of the form.

$$d\omega/dt = - B\rho v\omega \qquad (10)$$

where ρ is the ratio of air density to normal, where v is the remaining velocity of the projectile, and where

$$B = C_A d^4/A \qquad (11)$$

[1] Phil. Trans. Roy. Soc. A 221, pp. 295-387 (1920).

In (11), d is the diameter of the projectile in inches, A is axial moment of inertia of the projectile in lb-ft², and C_A is the axial couple coefficient. C_A was determined by R. H. Kent[1] to be 1.74×10^{-8} in this system of units. However, he used a smooth-headed projectile, while the M43A3 fuze has a channel cut into its outer surface for use in fuze setting. Theoretically, C_A should in general be a function of various dimensionless parameters such as $v/d\omega$, v/a_o, $d\omega/a_o$, (a_o being the speed of sound), Reynold's number, etc. but at the present time nothing is known about the form of the dependence of C_A on these parameters. In particular, C_A must depend to some extent upon the form of the projectile, and thus equations (10) and (11) cannot be strictly true with C_A a universal constant for they leave out of account such factors. In particular, one should expect that B for the M43A3 fuze should be somewhat greater than the value that is predicted by equation (11) when the value of C_A is used that was found by Kent. It is best to rely upon equation (11), with Kent's value of C_A, for a preliminary estimate of B and to determine a final value from the firings.

If one integrates (10) one finds at once that

$$\omega = \omega_o e^{-B\int_o^t \rho v dt} \tag{12}$$

where ω_o is the value of the spin, ω, at the muzzle in any convenient units. We shall use hundreds of revolutions per second. Thus, by (9),

$$t-\theta = -a''-b't-d'\omega_o^2 f(\theta) e^{-2B\int_o^t \rho v dt} \tag{13}$$

$$-c'\omega_o^2\int_o^t e^{-2B\int_o^{t'} \rho v dt} dt'$$

Since $t-\theta$ is small, and since the unknown constants are to be determined in any event from the firings so as best to represent them, it is permissible to replace $\underline{t}$ by θ in the right-hand member of the last equation. No serious error will thereby be introduced, if we write

$$t-\theta = a + b\theta - d'\omega_o^2 D(\theta) + c\omega_o^2 F(\theta) \tag{14}$$

[1] Ballistic Research Laboratory Report No. 154, 1939.

where a,b,c are new constants slightly different from $-a''$, $-b'$, $-c$ respectively, and where

$$d(\theta) = f(\theta)\, e^{-2B\int_0^\theta \rho v dt} \qquad (15)$$

$$F(\theta) = \int_0^\theta e^{-2B\int_0^{t'} \rho v dt}\, dt' \qquad (16)$$

In equation (14), a describes the zero-point error of the fuze, b the rate of the fuze at some constant spin, the d' term is the elastic strain term, and the c term is the effect of loss of spin. The integrals $D(\theta)$ and $\overline{F}(\theta)$ can be evaluated from the trajectory, if B is known; they depend for any given projectile on the air density at the ground, on the angle of departure, and on the muzzle velocity. In practice, they should be computed for a standard trajectory, and for three or four other trajectories differing from the standard trajectory in air density, muzzle velocity, and angle of departure. The wind effect has been studied. For the 3-inch gun M3 it amounts to only $-0^s.009$ at 30 seconds for a range wind of 40 miles per hour. There should be no cross wind effect to speak of, so wind effects can be ignored. It is sufficient to tabulate the integrals at multiples of five seconds in the fuze setting, θ, and then one can easily interpolate to find $D(\theta)$ and $F(\theta)$ for non-standard trajectories, range firings being at fuze settings that are multiples of five seconds. The initial spin, ω_o, is readily determined from the muzzle velocity in feet per second, v_o, the caliber d in inches, and the twist, one turn in s calibers, of the rifling:

$$\omega_o = 12\, v_o/100\, s \cdot d. \qquad (17)$$

Thus if the constants a,b,c, and B are known it is easy to evaluate the fuze error from equation (14) under all conditions. As already mentioned, $d' = 0.0086$.

5. Determination of the Constants a,b,c,B From 3-Inch Firings.

On the night of September 11, 1941, 360 3-inch rounds were fired at the Aberdeen Proving Ground to determine the fuze constants. The fuzes were all M43A3's, manufactured by the Frankford Arsenal; two guns were used. Gun No. 1 was a 3-inch AA gun M3, having a twist of rifling of 1 turn in 40 calibers. Gun No. 2 was a 3-inch AA gun M1918M1 having a twist of rifling of 1 in 25. The projectiles were all shell M42.

In order of time, the intended program was:

TABLE I

Group	No. Rds.	Gun	ft/sec Muzzle Velocity	Elevation	Fuze Setting
1	10	1	2600	45°	2^s
2	20	2	2600	45	2
3	10	1	2600	45	2
4	10	1	2600	45	15
5	40	2	2600	45	15
6	10	1	2600	45	15
7	10	1	2600	45	30
8	40	2	2600	45	30
9	10	1	2600	45	30
10	10	1	2800	45	5
11	20	1	2400	45	5
12	10	1	2800	45	5
13	10	1	2800	45	10
14	20	1	2400	45	10
15	10	1	2800	45	10
16	10	1	2800	45	30
17	20	1	2400	45	30
18	10	1	2800	45	30
19	20	1	2700	75	30
20	40	1	2700	25	30
21	20	1	2700	75	30

This program was strictly adhered to; but some rounds burst above clouds so that their times to burst could not be measured, and the velocities of course were not exactly equal to those intended. Times to burst were measured by the photelectric fuze chronograph; air densities were measured by means of radio-meteorograph balloons; velocities were measured for the five charges before and after firing, and interpolated to the round number of each gun separately. The velocities were corrected for differences of powder temperature between the times of firing and of the measurements of velocities. The radio-meteorograph records furnished, for each balloon ascent and at various heights, the ratio of actual air density to standard air density. By standard air density at any altitude, y, above the gun is meant the density given by the expression

$$0.07513\ e^{-hy} \text{ pounds per cubic foot.} \qquad (17a)$$

where y is the altitude in feet above the gun and where h=0.0000316 per foot.

per foot. The observed ratio of air density to standard depended to some extent upon the altitude of the balloon, and an exact allowance for this variation would have involved lengthy computations. Instead, a constant ratio of air density to standard for all altitudes was adopted in the computations, the value of the ratio being the average value, of the observed and slightly variable ratio, up to an altitude of 5000 meters. The dependence of time-to-burst upon air density being small, this simplification still yielded sufficiently accurate results. Since the average observed ratio depended upon the time of the ascent, it was interpolated to the time of firing of each round. Averaged according to the conditions of firing, the results were:

TABLE II

TABLE II

Line	Gun	φ_o	v_o ft/sec	N	ρ	θ sec	Mean Time to Burst t, sec.	s.d. of t, sec.	$W^{\frac{1}{2}}$	Wted. Resids. (20) sec.	Resids. (21) sec.
1	1	45°	2584	20	1.010	2	1.94	.0091	8	+.01	-.008
2	2	45	2560	20	1.010	2	1.75	.0091	8	-.04	——
3	1	45	2583	19	1.012	15	15.14	.026	3	-.02	+.004
4	2	45	2558	37	1.012	15	14.34	.025	3	+.02	——
5	1	45	2581	19	1.016	30	30.56	.040	2	+.06	+.011
6	2	45	2557	35	1.016	30	29.24	.039	2	+.06	——
7	1	45	2800	18	1.018	5	4.90	.015	5	-.14	-.014
8	1	45	2416	20	1.018	5	5.01	.0093	8	+.14	+.011
9	1	45	2797	20	1.019	10	9.98	.016	4	-.03	+.025
10	1	45	2412	20	1.019	10	10.08	.023	3	-.04	-.017
11	1	45	2795	20	1.020	30	30.34	.036	2	-.14	-.043
12	1	45	2411	20	1.020	30	30.70	.038	2	+.15	+.018
13	1	75	2664	40	1.022	30	30.40	.026	3	-.09	-.013
14	1	25	2664	38	1.022	30	30.56	.034	2	+.03	-.007

Here φ is the angle of elevation, v_o is the muzzle velocity, N is the number of observed rounds, ρ is the ratio of air density to standard, θ is the fuze setting, t is the mean time to burst, and "s.d." is the standard deviation, of the mean time to burst, estimated from within groups. Later columns will be explained later.

In equation (11), d is 3, and A is 0.1105 lb-ft². Using Kent's value of C_A in equation (11), one finds that B = 1.275×10^{-5}. By equation (17), for the M3 gun $\omega_o = v_o/1000$; for the 1918 gun $\omega_o = 0.0016\, v_o$. For reasons already mentioned, the preceding value of B may be in need of correction. Let the correction to B be $-\Delta$, so that the correct value of B is $1.275 \times 10^{-5} - \Delta$. In equation (14), B appears in the term involving $D(\theta)$, which is therefore altered by a change in B;

but the term is small and the alteration can be ignored when finding Δ. In the final computations, of course, the complete effect of a changed B will be allowed for. The main effect of a change in B is in the term involving $F(\theta)$, and one has for the increase in $F(\theta)$, when B is diminished by Δ, the quantity $\Delta \cdot J(\theta)$ where

$$J(\theta) = 2\int_0^\theta \left(e^{-2B\int_0^{t'} pvdt} \int_0^{t'} pvdt\right) dt' \qquad (18)$$

Thus (14) may be written,

$$t-\theta = a + b\theta - d'\omega_0^2 D(\theta) + c\omega_0^2 F(\theta) + \Delta \cdot c\omega_0^2 J(\theta). \qquad (19)$$

In this equation $D(\theta)$, $F(\theta)$, and $J(\theta)$ are to be evaluated for the uncorrected value of B, namely 1.275×10^{-5}. The quantities $D(\theta)$, $F(\theta)$, and $J(\theta)$ were obtained by numerical integration from the five trajectories:

Trajectory	v_0	φ	ρ
1	2600	45°	1
2	2800	45°	1
3	2800	45°	0.9
4	2700	400 mils	1
5	2700	1100 mils	1

The values of the integrals were then found for any particular set of values of v_0, φ, and ρ by linear interpolations for v_0 and ρ, and by second-order interpolations in the sine of φ. The quantities D, F, and J should vary only slightly with φ when φ is near 90°, so that an expansion of each of them in powers of $\sin \varphi$ is reasonable. The interpolations appeared to be more than sufficiently accurate when carried out as above described. The fourteen lines of the experimental table then yielded fourteen observational equations of the type (19), in the unknowns a, b, c, and $c\Delta$. These were solved by least squares. From $c\Delta$ and c the correction $-\Delta$ to B was determined. New values of the functions D, F, and J were computed from the new B. The least squares solution was then carried out again and yielded the value $B = 1.87 \times 10^{-5}$. In all these solutions, weights were adopted inversely proportional to the squares of the standard deviations. The square roots of the adopted weights appear in the tenth column, headed $W^{\frac{1}{2}}$, of Table II.

A third least squares solution was carried out, starting with the value $B = 1.87 \times 10^{-5}$. This time it was found that $10^5\Delta = -0.14 \pm 0.33$ (s.d.). Since the correction was so small compared to its standard deviation, it was not considered necessary to revise B yet again, and the value $B = 1.87 \times 10^{-5} \pm 0.33 \times 10^{-5}$ for the 3-inch shell, M42, was adopted as definitive. A further revision of B, by applying the trivial correction $-\Delta$, would have altered neither the standard deviation of B, nor the goodness of agreement of equation (14) with the observations, appreciably. The least squares value of B is of course related to the least squares value of c. If B is increased, so as to predict a greater rate of loss of spin, and c is appropriately diminished so as to result in a smaller dependence of rate on spin, then the goodness of agreement with the observations is not appreciably altered. The inclusion of the 1918 gun, with its large twist, helps to reduce the indeterminacy, but not to eliminate it altogether.

The third least squares solution was therefore simplified to a solution for three unknowns, namely a, b, c with Δ set equal to zero. The resulting values were

$$
\begin{aligned}
B &= 1.87 \times 10^{-5} \pm .33 \times 10^{-5}, \\
a &= -.037 \pm .015, \\
b &= .0479 \pm .0016, \\
c &= -.00741 \pm .00062, \\
C_A &= 2.55 \times 10^{-8},
\end{aligned}
\qquad (20)
$$

where the standard deviations were computed, however, from the residuals and from the weights in the 4-unknown solution so that the standard deviations are fair. The weighted residuals arising from the values (20) appear in the 11th column of the experimental table, in the sense $W^{\frac{1}{2}}$(observed t minus computed t). The standard deviation of an observational equation of weight unity, estimated from the residuals with ten degrees of freedom is 0.101 seconds. The best least squares solution, with Δ not fixed, gave instead 0.099, not appreciably smaller. From the choice of weights, the expected standard deviation of an observed time of weight unity is 0.074 seconds, estimated from many degrees of freedom. The probability of a larger disagreement by chance is given by the X^2 test with $X^2 = 10\,(.101)^2/(.074)^2 = 18.63$, for ten degrees of freedom, and is 0.047. Thus the agreement while perhaps tolerable is not very good, and it desirable to obtain constants that represent the observation more satisfactorily. It will be noticed from the experimental table that the largest weighted residuals occur in the firings at high and low velocities, in lines 7 through 12. The theory predicts too long a time for the high velocities, and this suggests that c should be numerically larger or that B should be larger. Neither change can be made, however, in view of the least squares results, without impairing the agreement for the 1918 gun; and if all the data are to be represented as well as possible, then the preceding least squares results are the best obtainable.

At muzzle velocities of 2600 feet per second, the spins for the 1918 and for the M3 gun are 416 and 260 revolutions per second, respectively. Thus the c term in (14) is 2.56 times as large for the 1918 gun as for the M3 gun. It is possible that over so wide a range of spins, the formula (6) is not sufficiently accurate to represent the dependence of the rate upon the spin. Rather than to include additional terms in the fourth power of the spin, thereby introducing another unknown and leading to very inaccurate results, it has seemed preferable to the writer to adopt the preceding results insofar as they relate to B, but to vary a, b, and c so as to get the best agreement for the M3 gun. The 1918 gun, with its high twist, is primarily a fuze-testing gun, and for fuze-cam data, results are needed that agree with the firings of the M3 gun, and with other guns producing normal spins. The spin produced by the 1918 gun is higher than that produced by any gun in which the M43A3 fuze is to be used by the Army. Thus the firings described in lines 2, 4, and 6 have been useful in finding B; but B having been determined as well as possible, the next thing to do is to represent the results observed with the M3 gun as well as possible. Ruling out, therefore, lines 2, 4, and 6 of the experimental table, a fourth least squares solution was made for a, b, and c with B set equal to 1.87×10^{-5} (so $C_A = 2.55 \times 10^{-8}$).

The results of this final solution were as follows:

$$
\begin{aligned}
a &= -0.014 \pm .023 \text{ (s.d.)} \\
b &= 0.0586 \pm .0032 \text{ (s.d.)} \qquad (21) \\
c &= -0.01025 \pm .00082 \text{ (s.d.)}
\end{aligned}
$$

where the standard deviations have been estimated from eight degrees of freedom on the assumption that B is known. The unweighted residuals observed t minus computed t appear in column twelve of the experimental table, the computed values being computed from equation (14) with the values of the constants given in (21). Making use of the weights, it is found from the residuals that the standard deviation of an observed time of weight unity is 0.071 seconds (from eight degrees of freedom). This is in excellent agreement with the expected standard deviation 0.074 seconds, the former being an externally estimated value and latter an internally estimated value. The probability of a larger value than 0.071 being obtained by chance is determined, from $X^2 = 7.36$, to be approximately 0.5.

The values of the a, b, c given by (21) are not really as accurate as the standard deviation there given, for B is not accurately known. However, since any error in B has been compensated for by an error in c so as still to represent the observations, the standard deviations given in (21) should give a fair idea of the goodness of agreement to expect between equation (14) and the observed times to burst, when the constants (21) are used in evaluating (14).

6. The Control of the Fuze Constants in Manufacture, and a Suggestion for an Improved Mechanical Time Fuze.

The constant a, the zero-point constant, is determined by the value of the initial elastic torque in the fuze, and by the manner in which the zero point of the fuze is adjusted during manufacture.

The constant b is determined by the rate to which the fuze has been regulated, and by the spin during regulation. The fuze is regulated at a value of ω equal to 2.7, in such a way that it vibrates 172.839 times per second, while 172.265625 vibrations correspond mechanically to one fuze second. Thus at a spin of 2.7,

$$\frac{dt}{d\theta} = 0.99668.$$

During flight, when the instantaneous spin is ω, one should have

$$\frac{dt}{d\theta} = 1 + b + c\omega^2 \tag{22}$$

and when ω is 2.7, and the values of b and c in (21) are used, equation (22) yields

$$\frac{dt}{d\theta} = 0.98388,$$

with an uncertainty (standard deviation) of only about 0.0068. If no change were produced in the properties of the escapement by firing, and if the rate in flight at a spin of 2.7 were the same as the rate in the regulating chuck at that spin, the two preceding values of $dt/d\theta$ ought to be the same; whereas they differ actually by 1.9 standard deviations, an amount somewhat too large to be reasonably attributed to chance. If the values (20) of b and of c are used instead of the values (21), the rate in flight at a spin of 2.7 is

$$\frac{dt}{d\theta} = 0.99388$$

with a standard deviation of about .0048, agreeing with the value 0.99668 to within 0.6 standard deviations, and tending to confirm the values of a,b,c given in (20), instead of the revised values given in (21). However, nothing is known about the effect of firing upon the rate of the fuze, which may be systematically altered by the enormous accelerations encountered in the gun. Moreover, when the projectile is in flight, lack of perfect axial symmetry must in general cause it to rotate about an axis other than the axis of the fuze escapement. There are physical reasons, therefore, for believing that the rate during flight need not be the same as during regulation in the regulating chuck, even at equal rates of spin. Finally, the values (21) represent the observed behavior of the fuze in flight much better than do the values (20), on the M3 gun. As we shall see later the values given by (21) lead to a better agreement with the observed time-to-burst, fuze-setting relation than do the values given by (20).

Although b need not correspond exactly to the rate to which the fuze is regulated, any change in the rate of regulation should result in an equal change in b. We see from (14) that increasing b causes all the fuze errors $t-\theta$ to be increased by an amount proportional to the fuze setting, θ. The adoption by the manufacturer of a standard frequency, higher by 1% than the recent standard, should decrease all values of $t-\theta$ by $0.01\ \theta$. At thirty seconds, this would amount to 0.30 seconds. Thus the manufacturer has it within his power easily to modify the time-to-burst, fuze-setting relation in the following way, for all guns and under all conditions:

Fuze setting, θ, in seconds:	0	5	10	15	20	25	30
Decrease in $t-\theta$, in seconds, caused by a 1% increase in the frequency to which the fuze is regulated	0	0.05	0.10	0.15	0.20	0.25	0.30

The writer is aware of no manner in which the manufacturer can control the value of c. The constant c, expressing the dependence of rate upon spin, appears to be intrinsic in the design of the fuze.

If c, as seems likely, arises from the dependence of the stiffness of the hair-spring upon the spin of the fuze, it should be possible to make it vanishingly small by designing a fuze with a different type of escapement. The balance wheel and escape wheel could be mounted as at present; but the transverse hair-spring in flexion should be replaced by an axial rod or thick wire in torsion. The torsional element could be one arm of the balance staff which, projecting through a sleeve bearing, and beyond the bearing for some distance, could be clamped firmly at its outermost end. The twisting of this arm of the balance staff would supply the restoring couple to yield an isochronous escapement, whose rate would be almost completely independent of the rate of spin. Regulation of the rate could be accomplished either (1) by balance screws, (2) by filing the torsion member to weaken it, (3) by sliding the clamp along the torsion member, or (4) by providing the torsion member with a sliding collar which, after being brought to the right position on the torsion member, could be fastened with a set-screw. A mechanical time fuze so constructed could be regulated to yield a time to burst that was always equal to the fuze setting, regardless of the rate of spin; and thus such a fuze would behave in the same manner in different guns, at all elevations, air densities, and muzzle velocities. There would be no differential effects.

7. The Relation of Time-to-Burst and Fuze-Setting, 3-Inch A.A. Gun M3.

7a. The Computed Time-to-Burst, Fuze-Setting Relation.

The experimental value 0.0086 for d', the value $B = 1.87 \times 10^{-5}$, and the values of a, b, and c given by (21), when inserted in equation (14), yield the following relation between time to burst, t, and fuze setting, θ, when the air density is standard, when the muzzle velocity is 2700 feet per second, and when the angle of elevation is 45°:

θ sec	$t-\theta$ sec
0	-.051
5	-.064
10	-.012
15	+.076
20	+.184
25	+.310
30	+.447

The quadratic expression in θ,

$$t-\theta = -.0653 + .00086\,\theta + .000552\,\theta^2 \qquad (23)$$

represents the preceding computed results with a maximum error of 0.017 seconds. The expression (23) was obtained by least squares. The cubic,

$$t-\theta = -.0533 - .00714\,\theta + .001272\,\theta^2 - .000016\,\theta^3, \qquad (24)$$

represents the computed results with a maximum error of .005 seconds.

7b. The Standard Time-to-Burst, Fuze-Setting, Relation; 3-Inch Gun M3

Besides the firings of September 11, 1941, which have been employed to find the fuze constants B, a, b, and c, other firings were carried out on August 8, September 8, and September 16, 1941. All of these were from the 3-inch A.A. gun M3, with shell M42 and fuzes M43A3. These three other firings were not sufficiently homogeneous with respect to the firing on September 11 for it to have been desirable to include them, along with the September 11 firings, in obtaining the fuze constants for differential effects. The inhomogeneity, evident on plotting the results, may have been the result of differences of a statistical nature between different lots of fuzes. For finding the fuze constants homogeneous data are essential, so only the very extensive September 11 firings were so employed. However, for finding a standard time-to-burst, fuze-setting relationship, the other firings have been included. The results were then adopted as standard, for inclusion in the recently prepared specifications for the M43A3 fuze. By means of the differential effects shortly to be tabulated, the fuze errors $t-\theta$ were all reduced to the following standard conditions: muzzle velocity 2700 feet per second, angle of elevation 45°, standard air structure. The air structure is standard when the densities at all altitudes y above the gun are given by equation (17a).

TABLE III; 3-Inch A.A. Gun M3

Date	No. Rds.	Fuze Setting θ seconds	Fuze error $t-\theta$, seconds	$W^{\frac{1}{2}}$
8-20	8	5	-.10	3
8-20	9	10	-.04	2
8-20	9	15	-.05	1
8-20	9	20	+.20	2
8-20	10	25	+.29	1
8-20	28	30	+.42	1
9-8	30	5	-.05	5
9-8	30	15	-.02	2
9-8	40	30	+.48	2
9-11	20	2	-.07	6
9-11	18	5	-.09	3
9-11	20	10	.00	3
9-11	19	15	+.09	2
9-11	117	30	+.44	4
9-16	40	6	-.01	4
9-16	20	16	-.05	2
9-16	40	30	+.47	2

In the firings of September 11, those rounds have here been omitted which were fired at 2400 feet per second, since correction to the standard velocity of 2700 feet per second, from 2400 feet per second, would not yield accurate results. The last column contains the square roots of weights, the weights being inversely proportional to the squares of the standard deviations (internally estimated) of the groups of rounds. Now if all the firings were homogeneous, weights so chosen should be used in combining the results. On the other hand, the assignment of equal weights to all groups would be preferable with inhomogeneous data, from the point of view of getting a fair average of different lots of fuzes, although such a choice of equal weights would leave out of account altogether the differing internal consistencies on different days and at different fuze settings. Therefore a compromise was adopted; the numbers in the last column of Table III, rather than their squares, were adopted as weights for combining results from different days at equal fuze settings. The adopted system of weights was thus intermediate between equal weights, and the set of weights W based on internal consistency, and thus made some allowance for differing internal consistencies among the groups being averaged. After the groups for different days, at each fuze setting, had been so combined the results were:

TABLE IV: 3-Inch A.A. Gun M3

Fuze Setting θ, seconds	Fuze error $t-\theta$, seconds	$W^{\frac{1}{2}}$	Residual Seconds
2	-.07	6	-.003
5	-.07	11	-.005
6	-.01	4	+.052
10	-.02	5	+.014
15	+.02	5	-.012
16	-.05	2	-.100
20	+.20	2	+.065
25	+.28	1	+.010
30	+.45	9	+.000

Column three contains the sum of the weights adopted for combining the data.

It was found in section 7a that a quadratic expression in θ represented the theoretical time-to-burst, fuze-setting relationship with a greatest error of only 0.017 seconds in $t-\theta$. A quadratic

$$t-\theta = \alpha + \beta\ \theta + \gamma\,\theta^2 \qquad (24')$$

was therefore fitted to the data of Table IV by least squares.

The unknowns, for convenience, were 10α, 100β, and 1000γ. In order to give proper allowance to the differing accuracies of the observational fuze errors at different fuze settings, in column (2) of Table IV, weights were adopted in the least squares solution equal to the squares of the numbers in column 3. The normal equations were

10α	100β	1000γ	=	$t-\theta$
3.1300	3.9970	8.8019		2.6180
	8.8019	23.7925		10.6745
		68.5153		33.0837

and the resulting values of the unknowns were

$\alpha = -.05924 \pm .023$ (s.d.)

$\beta = -.004761 \pm .0044$ (s.d.)

$\gamma = +.0007243 \pm .00013$ (s.d.)

so that equation (24) became

$$t-\theta = -.05924 - .004761\,\theta + .0007243\,\theta^2 \qquad (24'')$$

Equation (24) was then adopted as the standard time-to-burst, fuze-setting relation, under the standard conditions of 2700 feet per second muzzle velocity, 45° elevation, and normal air density, and was so incorporated in the specifications then being prepared, by the Laboratory, for the M43A3 fuze.

The standard time-to-burst, fuze-setting relation (24) has the following numerical values

Fuze Setting, θ, seconds	Fuze Error, $t-\theta$, seconds
0	-.059
5	-.065
10	-.034
15	+.032
20	+.135
25	+.274
30	+.450

and the differences between the observed fuze errors in Table IV, and the standard values given by (24), are tabulated in the last column of Table IV in the sense observed <u>minus</u> standard.

The standard relation is of a purely observational character, is plotted in Figure 1, and does not agree precisely with the theoretical relation tabulated in section 7<u>a</u>. The observed values of $t-\theta$, from column 2 of Table IV, are plotted in Figure 1 and so also is the theoretical relation.

The standard deviations of the observational constants α, β, and γ are estimates obtained in the usual manner from the residuals of the least squares solutions, and correspond to six degrees of freedom. These constants do not differ significantly from the constants in the theoretical parabola (23), for the differences, divided by the standard deviations of the observational constants, are

$$(.0653-.0592)/.023 = +.27$$

$$(.00086-.00476)/.0044 = -1.28$$

$$(-.00055+.00072)/.00013 = +1.31$$

It is known[1] that the ratio, of the error of any least squares value to the estimate of its standard deviation obtained in the usual manner from the residuals, is distributed in accordance with the t-law with a number of degrees of freedom equal to the number of observational equations diminished by the number of unknowns. If the theoretical constants were correct, the observational values would give rise to ratios numerically larger than those observed (+.27, +1.28, and +1.31) in 80%, 25%, and 24%, respectively, of such cases. Thus the standard relation does not differ significantly from the theoretical relation of section 7a.

The theoretical relation followed from the values of the fuze constants (21). These values were adopted in preference to the values (20), which gave the best over-all agreement when the 3-inch A.A. gun M1918M1 was considered along with the gun M3. The theoretical relation that would result from the adoption of the fuze constants (20) instead of (21) is represented in Figure 1 as a dotted curve. It will be seen that it does not agree with the standard, purely observational, relation as well as does the theoretical relation of section 7a.

8. Differential Effects for the 3-Inch A.A. Gun M3.

8a. Effect of a Change in Muzzle Velocity.

A change in muzzle velocity changes ω_0 and the trajectory, and therefore $D(\theta)$ and $F(\theta)$. By equation (14), the values of t have been computed for muzzle velocities of 2600 and 2800 feet per second, at 45° and standard air density. The differential effect of an increase of 100 feet per second upon the time to burst, t, has thus been found to be:

[1] Sterne, Proc. Nat. Acad. Sci., 20, pp. 565-571 (1934).

Fuze setting θ; seconds	Decrease in t caused by an increase of 100 ft/sec in muzzle velocity; seconds.
0	.003
5	.024
10	.040
15	.054
20	.066
25	.076
30	.085

8b. Effect of a Change in Air Density, 3-Inch Gun M3

Increasing the air density increases the rate of loss of spin, and increases the time to burst, t. The effect has been computed from equation (14), using trajectories for standard air density and nine-tenths of standard, at 45° and at 2800 feet per second. The results would be nearly the same at 2700 feet per second, and are:

Fuze setting, θ; seconds	Increase in t caused by an increase of 1% in air density; seconds
0	.0000
5	.0006
10	.0015
15	.0025
20	.0034
25	.0045
30	.0056

As has been mentioned in section 5, standard air density is a function of altitude above the gun, and is defined by equation (17a). An increase of 1% in air density refers to an increase of 1% with respect to standard density, at all altitudes. If the actual air structure is known, and if the percentage of increase over standard is different at different altitudes, then an average of the actual percentage excess over standard, from the gun to the altitude of burst, will furnish very accurate results when used in conjunction with the preceding table. If the air structure is unknown, the table will furnish less accurate, but still useful, results when employed in conjunction with the air density at the gun. In that case the percentage, by which the air density at the gun exceeds the value 0.07513 pounds per cubic foot, should be adopted as the percentage increase of air density over standard.

8c. Effect of a Change in Angle of Elevation, 3-Inch Gun M3.

Increasing the angle of elevation moves the trajectory into regions of lower air density, diminishes the rate of loss of spin, and shortens the time to burst. The effect has been computed from equation (14), using trajectories at standard air density, 2700 feet per second muzzle velocity, and at angles of elevation of 400 mils, 800 mils, and 1100 mils. Denote by $t(\theta,\varphi)$ the time to burst at an angle of departure (for practical purposes, angle of elevation) φ. Then the differences

$$\lambda(\theta,\varphi) = t(\theta,\varphi) - t(\theta,45°)$$

have been computed for values of φ of 400 and 1100 mils. It was found that $\lambda(\theta,400)$ and $\lambda(\theta,1100)$ could each be represented by a quadratic function of θ, with a greatest error of 0.0006 seconds. The directly computed values were therefore smoothed[1] by replacing them by those quadratic expressions, computed by least squares. It was further found that at any fuze setting, θ, $\lambda(\theta,\varphi)$ could be represented by linear function of sin φ,

$$\lambda(\theta,\varphi) = A''(\theta) + b'(\theta) \sin\varphi \qquad (25)$$

determined by least squares, with a greatest error of only 0.0004 seconds. Therefore A and B' were determined for each fuze setting by solving the observational equations

$$A''(\theta) + B'(\theta) \sin 400 \text{ mils} = \lambda(\theta,400)$$

$$A''(\theta) + B'(\theta) \sin 45° = 0$$

$$A''(\theta) + B'(\theta) \sin 1100 \text{ mils} = \lambda(\theta,1100)$$

by least squares for $A''(\theta)$ and $B'(\theta)$, with the results:

θ	$A''(\theta)$	$B'(\theta)$
0	-.0003	+.0006
5	+.0060	-.0084
10	+.0246	-.0347
15	+.0553	-.0782
20	+.0982	-.1389
25	+.1534	-.2168
30	+.2207	-.3120

From these values of A'' and B', and equation (25), the following differential effects $\lambda(\theta,\varphi)$ were computed:

[1] Such smoothing, when practicable, is the best sort ever to use.

Time to burst, less time to burst at 45° elevation, in seconds

Fuze setting, seconds	Angle of elevation 0°	15°	30°	45°	60°	75°	90°
0	0.000	.000	.000	.000	.000	.000	.000
5	.006	.004	.002	.000	-.001	-.002	-.002
10	.025	.016	.007	.0005	-.005	-.009	-.010
15	.055	.035	.016	.000	-.012	-.020	-.023
20	.098	.062	.029	.000	-.022	-.036	-.041
25	.153	.097	.045	.000	-.034	-.056	-.063
30	.221	.140	.065	.000	-.049	-.081	-.091

From the way in which the values of A and B' were obtained, they must be quadratic functions of θ, and they do in fact have constant second differences, as do the values of $\lambda(\theta,\varphi)$ in the preceding table. A second-order interpolation in θ, and a linear interpolation in $\sin\varphi$, will yield the effect of changes of elevation for values of θ, and φ, not tabulated. Some of the longer fuze settings, at the lower elevations, although included in the preceding table, cannot occur in A.A. fire because rounds so fired would burst in the ground.

9. The Standard Deviation of the Time to Burst

The standard deviation of the time to burst at a given fuze setting, estimated from within a group of rounds fired under identical conditions with fuzes manufactured at the same time, increases with increasing fuze setting. The 3-inch firings which have been discussed lead to a linear relation between the standard deviation and the fuze setting, θ; the standard deviation increasing uniformly from 0.06 seconds at 5 seconds to 0.20 seconds at 30 seconds. The observed estimates are subject to statistical uncertainties that are too large to enable any significant deviation from linearity to be found. Denoting the standard deviation by σ, the straight line

$$\sigma = .032 + .0056\,\theta \qquad (26)$$

determined by least squares, represents the observed values correctly to within their statistical uncertainties.

10. The Relation of Time-to-Burst and Fuze-Setting, 90mm A.A. Gun M1, and the Differential Effects.

10a. Determination of the Constants.

On the nights of August 25 and 26, 1941, 200 rounds were fired from a 90mm A.A. gun M1, using shell M71 and M43A3 time fuzes, manufactured by the Frankford Arsenal. On both nights, the average ratio of air density to standard was 0.97. The results were as follows:

TABLE V. 90mm A.A. Gun M1

Date	Elevation, φ, mils	Muzzle Velocity, v_o, ft/sec	No. Rds. N	Fuze Setting θ, sec	Mean Time to Burst, t, sec	s.d. of t sec	$\sqrt{W}$	Residuals in seconds; t observed minus t computed
25	800	2716	15	5	4.81	.019	2	+.009
25	800	2676	15	10	9.81	.022	2	-.007
25	800	2709	15	15	14.80	.034	1	-.037
25	800	2682	15	20	19.91	.021	2	+.001
25	800	2690	13	25	25.07	.036	1	+.094
25	800	2700	23	30	30.03	.037	1	-.022
26	400	2668	15	5	4.80	.019	2	-.015
26	400	2628	15	10	9.89	.022	2	+.044
26	400	2660	15	15	14.83	.034	1	-.056
26	400	2634	15	20	19.94	.021	2	-.038
26	400	2642	15	25	25.08	.034	1	+.006
26	400	2652	25	30	30.22	.036	1	+.040

For the M71 projectile with the M43A3 fuze, A = .2753 lb-ft². The value $C_A = 2.55 \times 10^{-8}$ determined from the 3-inch firings, in conjunction with the preceding value of A, leads to $B = 1.46 \times 10^{-5}$. The twist of the gun is 1 turn in 32 calibers, so that by equation (17), $\omega_o = .0010583\, v_o$.

For reasons which have already been mentioned, the preceding value of B is uncertain. It would be theoretically possible to adopt the same value of c as for the 3-inch gun, since c should depend only on the fuze, and to adjust B, a, and b so as to represent the observations as well as possible. However, the same result can be achieved practically by holding B fixed at 1.46×10^{-5} and by varying all three of a,b,c. It is not perhaps physically necessary for b to be exactly the same for the 90mm gun as for the 3-inch, for the shells may wobble differently in flight; similarly, there may be reasons why the constant a (the zero point constant) may differ for the two guns. The theoretical question of the constancy of the "constants" is difficult; but practically, it is desirable to adjust a and b in order to represent the observed firings as well as possible.

The functions $D(\theta)$ and $F(\theta)$ were evaluated as for the 3-inch gun, and the observations in Table V yielded twelve observational equations of the type (14) from which the best values of a,b,c were determined by least squares. In the least squares solution, weights were adopted inversely proportional to the squares of the standard deviations of the mean times to burst; the square roots of these weights appear in the column headed $W^{\frac{1}{2}}$. The residuals of the least squares solution, in the sense observed t minus computed t, appear in the last column of Table V. The values of a,b,c found by least squares were:

$$a = -.099 \pm .043 \text{ (s.d.)},$$
$$b = +.0499 \pm .012 \text{ (s.d.)}, \qquad (27)$$
$$c = -.00874 \pm .0026 \text{ (s.d.)}.$$

In comparison with the values of the constants (21) found from the 3-inch M3 firings, the a for the 90mm gun appears to be smaller than the a for the 3-inch M3 gun. The amount of the difference is 0.085, and .085/.043 = 2.0. The standard deviations in (27) are estimates from nine degrees of freedom. If the a for the 3-inch gun M3 were free from error, the t-distribution shows that a ratio numerically larger than 2.0 would arise in about 8% of such comparisons, and the uncertainty of the 3-inch value increases this percentage. The disagreement is thus not of much statistical significance. The values of b and c in (27) are in good agreement with the values in (21), when account is taken of their standard deviations.

From the residuals of the least squares solution, the standard deviation of an observational equation of unit weight is found to be .058 seconds. This is an estimate obtained from nine degrees of freedom. The value, based on many degrees of freedom, expected from the choice of weights is .039. The probability of a larger estimate than .058, from nine degrees of freedom, is found from the X^2 distribution to be about .019. Thus the agreement of the observations with the most closely fitted theoretical relation, corresponding to the values (27), is not good; in only about 2% of such cases should chance give rise to such large discrepancies. An inhomogeneity between the two nights may be the reason for the disagreement.

10b. The Computed Time-to-Burst, Fuze-Setting, Relation, 90mm Gun M1.

From the constants (27), the time-to-burst, fuze-setting relation has been computed by equation (14), at 2700 feet per second muzzle velocity, standard air structure and 45° elevation:

Fuze-Setting, θ seconds	Time to burst less fuze setting, t-θ, seconds
0	-.141
5	-.195
10	-.193
15	-.159
20	-.104
25	-.031
30	+.053

The quadratic

$$t-\theta = -.1524 - .00839\,\theta + .000518\,\theta^2 \qquad (28)$$

represents the preceding computed results with a greatest error of .014 seconds, and the cubic

$$t-\theta = -.1424 - .01506\,\theta + .001118\,\theta^2 - .0000133\,\theta^3 \qquad (29)$$

with a greatest error of .004 seconds.

10c. The Empirical Time-to-Burst, Fuze-Setting Relation, 90mm Gun M1.

By means of the differential effects shortly to be tabulated, the fuze errors $t-\theta$ in Table V were all reduced to the standard conditions of 2700 feet per second, standard air structure, and 45° elevation. The averaged results were:

TABLE VI. 90mm A.A. Gun M1

Fuze setting θ, seconds	Fuze Error, $t-\theta$, seconds	W	Residuals, seconds
5	-.20	2	-.01
10	-.17	2	+.02
15	-.20	1	-.04
20	-.12	2	-.01
25	+.03	1	+.06
30	+.07	1	-.01

Since the theoretical relation was so well represented by a quadratic in θ, a quadratic was fitted to the observed $t-\theta$ by least squares, yielding

$$t-\theta = -.1665 - .00774\,\theta + .000536\,\theta^2 \qquad (30)$$

The residuals, observed $t-\theta$, less $t-\theta$ given by (30), appear in the last column of Table VI. Equation (30) is purely empirical in nature, and is plotted in Figure 2.

The standard deviations (estimates from three degrees of freedom) of the three constants in (30), obtained from the residuals, are respectively .052, .0078 and .00024. The constants in (30) differ from the constants in the theoretical relation (28) by much less than their standard deviations, so that there is no significant difference between the empirical and the theoretical relations. The theoretical relation agrees very nearly as well with the observations as the empirical. The disagreement reported in 10a, between the observations and the theory, is essentially no worse than the disagreement between observations and any smooth curve at all like (30), passed through the observational points; the trouble lies in the observations themselves, rather than the theory.

The close agreement of the empirical and theoretical curves is of course a consequence of the manner in which the "theoretical" curve was obtained. Since both the constants (27) and the coefficients in (30) were chosen so as to produce the best agreements with the same data, the theoretical and empirical relations could not differ appreciably. The fuze-cam data for the 90 mm gun, already supplied by the Laboratory, have been based on the theoretical relation.

10c. Differential Effects, 90mm Gun M1.

The effect of a change in muzzle velocity has been computed from equation (14) and is:

Fuze Setting, θ, seconds	Decrease in t caused by an increase of 100 ft/sec in muzzle velocity, seconds
0	.003
5	.024
10	.040
15	.054
20	.065
25	.074
30	.084

The effect of a 1% increase[1] in air density, computed from (14) is:

Fuze Setting, θ, seconds	Increase in t caused by 1% increase in air density, seconds
0	0
5	.0004
10	.0011
15	.0020
20	.0028
25	.0037
30	.0047

The effect of a change in angle of elevation,

$$(\theta,\varphi) = t(\theta,\varphi) - t(\theta,45°)$$

[1] The reader is referred to Section 8b for remarks on the meaning and use of the table.

is given by

$$(\theta,\varphi) = A''(\theta) + B'(\theta) \sin \varphi \tag{31}$$

where A" and B' have the values

θ	$A''(\theta)$	$B'(\theta)$
0	-.0006	+.0008
5	+.0069	-.0094
10	+.0246	-.0341
15	+.0526	-.0736
20	+.0911	-.1280
25	+.1398	-.1971
30	+.1992	-.2811

From equation (31), the following differential effects for elevation have been obtained:

Fuze setting, θ, seconds	Time to burst, less time to burst at 45° elevation, in seconds						
	Angle of elevation φ						
	0°	15°	30°	45°	60°	75°	90°
0	0	0	0	0	0	0	0
5	.007	.004	.002	0	-.001	-.002	-.003
10	.025	.016	.008	0	-.005	-.008	-.010
15	.053	.034	.016	0	-.011	-.018	-.021
20	.091	.058	.027	0	-.020	-.033	-.037
25	.140	.089	.041	0	-.031	-.051	-.057
30	.199	.126	.059	0	-.044	-.072	-.082

To interpolate, use second differences in θ, and first differences in φ.

The 4.7-Inch A.A. Gun: Preliminary Computations

Preliminary computations have been made of the expected time-to-burst, fuze-setting relation and of the differential effects for a 4.7-inch gun, with a standard muzzle velocity of 3000 feet per second, a projectile having an axial moment of inertia of $A = 1.072$ lb-ft², and a twist of either 1 turn in 25 calibers or of 1 turn in 30 calibers. The fuze is to use the M43A3 movement. There were only 28 rounds with observed times-to-burst, practically worthless for determining the constants a, b, and c or even a and b alone, and only rough preliminary estimates could be made. Rather than present the tentative computations, it is preferable to include here the exact evaluations of the quantities that will be needed, in the final solution with extensive range firings, for the time-to-burst, fuze-setting relation and differential effects. Such quantities as will be here presented can be used to expedite the final computations after extensive firing results are available.

With a twist of 1 in 25, the initial spin (by equation (17)) will be

$$\omega_o = .0010213\ v_o$$

and with a twist of 1 in 30, the initial spin will be

$$\omega_o = .0008511\ v_o.$$

Using $C_A = 2.55 \times 10^{-8}$, and the preceding value of A, one finds from equation (11) that in the English system of units B is 1.162×10^{-5}. It will be noticed in equations (15) and (16) that $D(\theta)$ and $F(\theta)$ both involve the exponential of a definite integral, which we may call $G(\theta)$,

$$G(\theta) = e^{-2B\int_0^\theta \rho v dt}$$

Thus

$$D(\theta) = f(\theta)G(\theta),$$

$$F(\theta) = \int_0^\theta G(t')\ dt'$$

For each of four trajectories, the quantities $G(\theta)$, $D(\theta)$, $F(\theta)$ have been evaluated, with $B = 1.162 \times 10^{-5}$ per foot, or $2B = 7.6214 \times 10^{-5}$ per meter. The four trajectories are

Trajectory	v_o ft.sec	Elevation mils	Ratio of Air-Density to Standard
1	3000	800	1
2	2800	800	1
3	3000	400	1
4	3000	1100	1

and the quantities $G(\theta)$, $D(\theta)$, $F(\theta)$ are

	Trajectory 1			Trajectory 2			Trajectory 3			Trajectory 4		
θ	G(θ)	F(θ)	D(θ)	G(θ)	F(θ)	D(θ)	G(θ)	F(θ)	D(θ)	G(θ)	F(θ)	D(θ)
0	1	0	.59	1	0	.59	1	0	.59	1	0	.59
$2\frac{1}{2}$	.861			.869			.857			.864		
5	.770	4.34	.59	.781	4.38	.60	.757	4.32	.58	.778	4.36	.60
$7\frac{1}{2}$	.706			.720			.683			.719	[illegible]	
10		7.89	.58		7.99	.59		7.75	.55		7.97	.60
$12\frac{1}{2}$	.621			.636			.580			.643		
15		11.01	.57		11.18	.58		10.67	.52		11.20	.59
$17\frac{1}{2}$	.568			.583			.512			.597		
20		13.85	.55		14.10	.56		13.23	.49		14.19	.58
$22\frac{1}{2}$	.531			.546			.462			.565		
25		16.51	.50		16.83	.51		15.54	.43		17.02	.54
$27\frac{1}{2}$	.502			.517			.421			.545		
30		19.02	.44		19.42	.45		17.65	.36		19.74	.48

Should any change finally be made in B, in the nature of a multiplication by some factor δ, the new G's can readily be obtained by raising the tabulated G's to the power δ. From the new G's, the new F's and D's can immediately be obtained.

11. The 75mm Aircraft Gun, M2. Predicted Time-to Burst, Fuze-Setting Relationship.

In accordance with a request from the Commanding General, Frankford Arsenal[1] time-to-burst, fuze-setting relationships were computed at various altitudes and air speeds for the 75mm Aircraft Gun, M2, firing Shell M48 equipped with M43A3 fuzes, at a muzzle velocity of 1850 ft/sec. There were no experimental fuze data for this gun, so the fuze constants (21) were used. The twist is one turn in 25.58 calibers, and the spin at the muzzle is given by (17). The axial moment of inertia of the projectile is 0.1260 lb-ft^2. Times to burst were to be computed only up to ten seconds; the trajectory being very flat, and the firing nearly horizontal, the y-component of motion could be ignored completely in the fuze computations. In evaluating D(θ) and F(θ), a constant drag coefficient was adopted in order that the integrations could be performed without numerical quadratures. Within the right-hand members of (15) and (16), the density ρ was taken to be constant and equal to the value appropriate to the altitude; v_0 was of course the sum of the true air speed, in feet per second, and 1850. From equation (11), and the the value $C_A = 2.55 \times 10^{-8}$, B was determined to be 1.538×10^{-5}.

[1] File FA 471.8261/2107; APG 471.821/423; letter dated January 15, 1942.

The results predicted from equation (14) were:

Altitude 3000 feet

Time to burst, seconds	Fuze setting in seconds at true air speed of 200 mph	300 mph	400 mph
2	2.10	2.10	2.10
4	4.14	4.14	4.14
6	6.16	6.16	6.15
8	8.17	8.17	8.16
10	10.17	10.16	10.15
	Altitude 15000 feet		
2	2.11	2.11	2.11
4	4.15	4.15	4.15
6	6.19	6.18	6.18
8	8.21	8.20	8.20
10	10.22	10.21	10.21
	Altitude 27000 feet		
2	2.11	2.12	2.11
4	4.16	4.17	4.16
6	6.21	6.21	6.20
8	8.25	8.24	8.23
10	10.28	10.26	10.25
	Altitude 39000 feet		
2	2.12	2.12	2.12
4	4.17	4.17	4.17
6	6.22	6.22	6.21
8	8.26	8.26	8.25
10	10.30	10.29	10.29

13. Summary.

a. An increase in the driving torque, at constant spin, makes the fuze run very slightly slower. An increase in the rate of spin makes the fuze run decidedly faster. The effect of spin is too large, and of the wrong sign, to be attributed to the effect of spin upon torque and of torque in turn upon the rate of the fuze. It is concluded that the effect of spin upon rate is probably through the hair-spring, whose tension is increased by an increased spin. The dependence of the rate upon spin has been determined from firings, and the change in rate appears to be nearly proportional to the change in the square of the spin, over the experimental range of spins. The fuze is caused to run slower by approximately one part in one hundred thousand by an increase of one degree Fahrenheit in its temperature, a temperature dependence so small as to be ignorable practically. There is some straining of the gear train by the driving torque, so that the diminishing driving torque, during flight, causes an untwisting of the timing disk relative to the escapement. This effect has been experimentally measured, and is almost small enough to be ignored. It introduces, however, a dependence of the zero-point of the fuze upon the initial elastic torque established during manufacture. The elastic torque should therefore be closely controlled.

b. Special firings of the M43A3 fuze from a 3-inch A.A. gun M3n, and from a 3-inch A.A. gun M1918 M1, have enabled the rate of loss of spin during flight, the zero point of the fuze, the dependence of rate upon spin, and the rate of the fuze in flight at a known spin, to be determined. From those quantities the time-to-burst, fuze-setting relation has been computed for the M3 gun for standard conditions: 2700 feet per second muzzle velocity, 45° elevation, and standard air structure. The effect upon the time-to-burst of departures from the standard conditions have also been computed. The time-to-burst, fuze-setting relation has also been obtained empirically from a large number of 3-inch firings, whose times-to-burst were reduced to the standard conditions. The computed relation is in substantial agreement with the empirical relation, and the empirical relation has been embodied in the specifications of the M43A3 fuze.

c. The fuze manufacturer cannot control the dependence of fuze rate upon spin; the amount of the dependence is intrinsic in the design of the fuze. He can control the zero point, altering it either directly or by changing the initial elastic torque of the fuze. He can also control the rate to which the fuze is regulated. If he increases the frequency to which it is regulated by 1%, he will diminish all times-to-burst, at all fuze settings and in all guns, by 1%.

d. It is theoretically possible to design a mechanical time fuze whose time-to-burst is equal to its fuze setting, under all conditions and in all guns. In such a fuze the transverse hair-spring, in flexion, of the present fuze must be replaced by an axial rod or wire in torsion.

e. In the 3-inch gun M3, the standard deviation of the time-to-burst, within a group of rounds fired under identical conditions, with fuzes manufactured at the same time by the Frankford Arsenal, has been determined. The standard deviation increases nearly linearly from approximately 0.06 seconds at five seconds to approximately 0.20 seconds at thirty seconds.

f. The time-to-burst, fuze-setting relation has been determined for the 90mm A.A. gun M1. for standard conditions: 2700 feet per second muzzle velocity, standard air structure, 45° elevation. The effect upon the time-to-burst of departures from standard conditions hase been computed.

g. Preliminary computations have been made of certain quantities needed in obtaining the time-to-burst, fuze-setting relations for the 4.7-inch A.A. gun. Pending extensive range firings, the computations are unfinished.

h. Time-to-burst, fuze-setting relationships have been predicted for the 75mm aircraft gun, M2, at various air speeds and altitudes. In the absence of experimental firings with the M43A3 fuze, the prediction is necessarily mathematical, but use has been made of the fuze constants experimentally determined from the 3-inch firings.

T. E. Sterne

T. E. Sterne,
Captain, Ord. Dept.

Appendix

Authority From Chief of Ordnance

C O P Y

00.471.821/1392 SF/mmp APG 471.821/380

September 2, 1941

Subject: Program of Firings to Provide Differential Effects in Mechanical Fuze, M43A3, for 3" M3 Gun.

To : Commanding General, Aberdeen Proving Ground, Maryland.

1. As a result of a conference held in this office on August 29, 1941, which covered 90m/m cam data and also the preparation of the specification for acceptance of the M43A3 Fuze, four hundred and sixty (460) M43A3 Fuzes are being furnished the proving ground for use in various tests in the 3" A.A. Gun M1918M1 and the 3" A. A. Gun M3.

2. One hundred (100) of these fuzes are to be fired for the purpose of augmenting data available for use in the preparation of the specification for the M43A3 Fuze in the 3" A. A. Gun, M3. The remaining three hundred and sixty (360) fuzes are to be used to obtain data on effect of spin on rate and also on effect of muzzle velocity and elevation on rate.

3. The Ballistic Research Laboratory at the proving ground will prepare the necessary program. It is requested that the firing of the program be expedited upon receipt of these fuzes in order that the specification can be completed. The proving ground is to prepare and submit the specification for study and approval.

4. No further authority for tests of the fuzes is required by this office.

By order of the Chief of Ordnance:

Arthur B. Domonoske
Lt. Col., Ordnance Dept.
Assistant

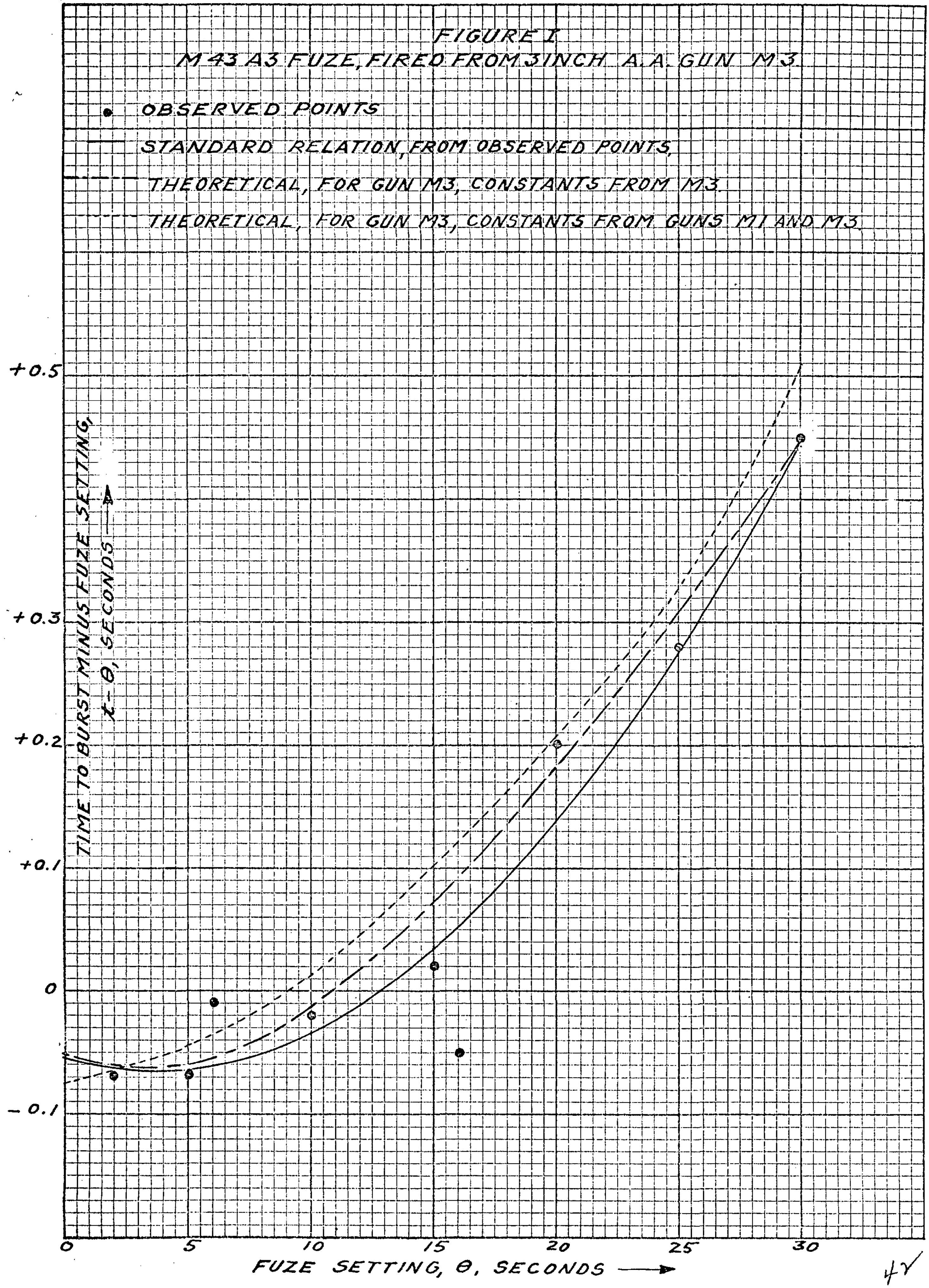


FIGURE I
M 43 A3 FUZE, FIRED FROM 3 INCH A.A. GUN M3
OBSERVED POINTS
STANDARD RELATION, FROM OBSERVED POINTS
THEORETICAL, FOR GUN M3, CONSTANTS FROM M3
THEORETICAL, FOR GUN M3, CONSTANTS FROM GUNS M1 AND M3
TIME TO BURST MINUS FUZE SETTING, t - θ, SECONDS
+0.5
+0.3
+0.2
+0.1
0
- 0.1
0
5
10
15
20
25
30
FUZE SETTING, θ, SECONDS

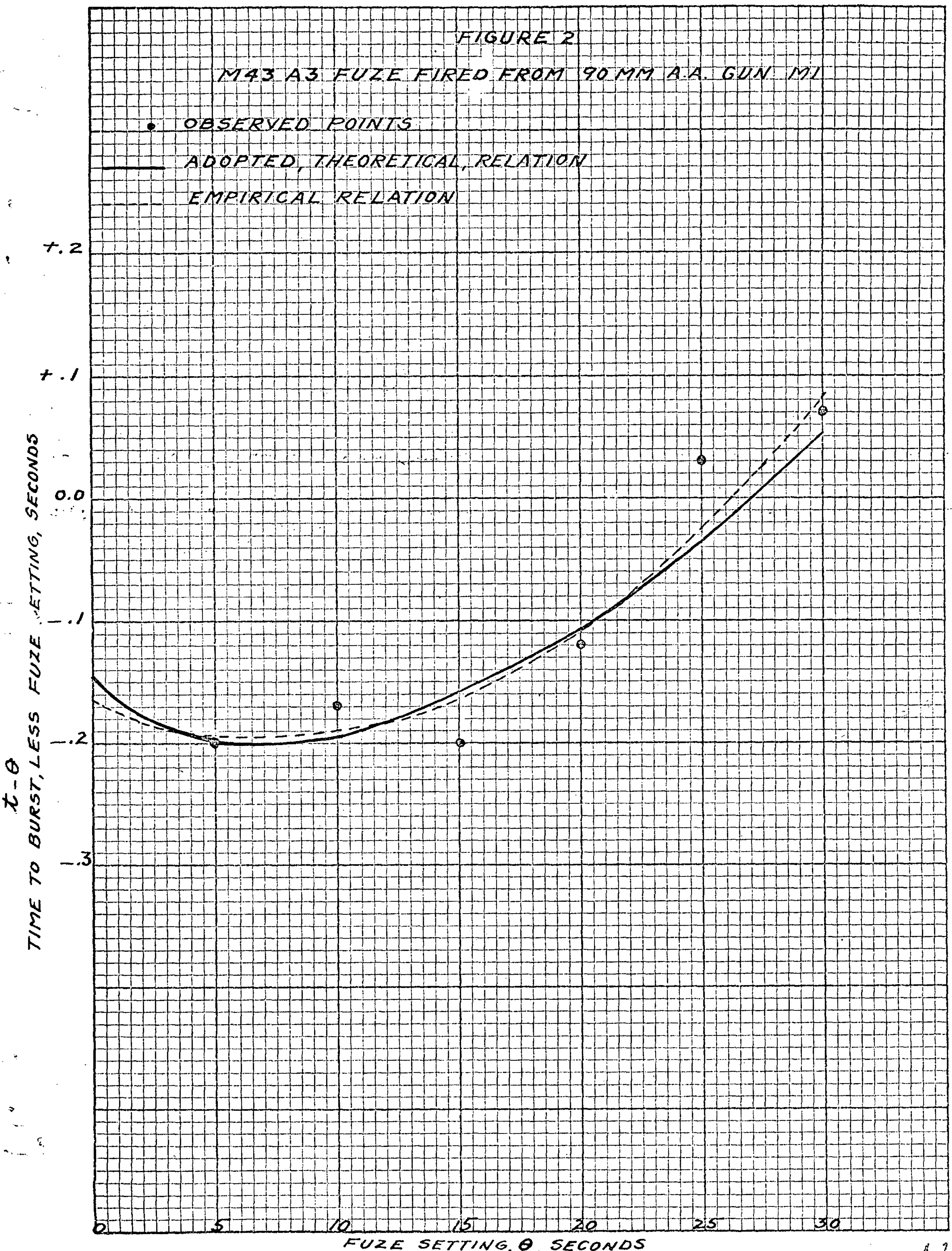


FIGURE 2
M43 A3 FUZE FIRED FROM 90 MM A.A. GUN M1
OBSERVED POINTS
ADOPTED, THEORETICAL, RELATION
EMPIRICAL RELATION
+.2
+.1
0.0
-.1
-.2
-.3
t - θ
TIME TO BURST, LESS FUZE SETTING, SECONDS
0
5
10
15
20
25
30
FUZE SETTING, θ, SECONDS